

Welfare Maximizing Fiscal and Monetary Policy Rules

Robert Kollmann (*)

Department of Economics, University of Bonn
24-42 Adenauerallee, D-53113 Bonn, Germany

Centre for Economic Policy Research, UK

March 30, 2003

This paper studies the welfare effects of monetary and fiscal policy rules, in a dynamic general equilibrium model with sticky prices. The model features capital accumulation and endogenous labor effort, and exogenous productivity shocks. Government purchases are valued positively by the private sector. These purchases are financed using a proportional income tax. The government issues nominal one-period bonds. Monetary policy is described by an interest rate rule; fiscal policy is described by rules according to which the income tax rate and government purchases are set as functions of GDP. Sims' (2000) quadratic approximation method is used to solve the model, and to compute household welfare. The paper determines the response coefficients of the policy rules that maximize household welfare. Optimized monetary policy has a strong anti-inflation stance; optimized fiscal policy implies that the income tax rate is countercyclical, and that government purchases are procyclical; this result does not hinge on the degree of price stickiness.

JEL classification:

Keywords: Monetary policy; Fiscal policy; Welfare.

(*) Tel.: 49 228 734073; Fax: 49 228 739100; E-mail: kollmann@wiwi.uni-bonn.de
<http://www.wiwi.uni-bonn.de/kollmann>

I thank Michael Devereux, Chris Erceg, Henry Kim and Jinill Kim for useful discussions.

1. Introduction

Much recent research has studied what monetary policy rules are best suited for stabilizing the economy and raising (private sector) welfare over the business cycle (see, e.g., Clarida et al. (1999), Taylor (1999), McCallum (1999), and Woodford (2003) for surveys of the relevant literature). The analysis of *fiscal* policy rules has received less attention, in the macroeconomics literature.

Existing contributions can be classified into two categories: (i) Dynamic extensions of Ramsey (1927) that determine time paths of fiscal policy instruments that maximize household welfare, subject to "implementability" conditions consisting of the private sector's decision rules and equilibrium conditions (see, e.g., Lucas and Stokey (1983)).¹ (ii) Studies that analyze the macroeconomic consequences of "simple" feedback rules for fiscal policy instruments (e.g., Taylor (2001)).

The Ramsey approach is appealing as it uses fully micro-based models and focuses on household welfare as the criterion for evaluating the effects of policy. However, solving dynamic Ramsey problems raises technical difficulties that are not yet fully resolved.² Also, existing studies that use the Ramsey approach focus on fairly stylized models.

Previous quantitative studies on the effect of "simple" fiscal policy rules have mostly used models that are not fully micro-based, and these studies use ad hoc criteria to evaluate policy rules (policies are evaluated according to the implied volatilities of output, inflation etc).

The present paper uses a business cycle model with rigorous micro-foundations. It focuses on "simple" policy rules, and it determines the response coefficients of those rules that maximize household welfare.

The model assumes a closed economy dynamic general equilibrium model with capital accumulation and endogenous labor effort, and exogenous productivity shocks. Government purchases are valued positively by households. These purchases are financed using a proportional income tax. The government issues nominal one-period bonds. There is monopolistic competition in goods markets, and goods prices are set in a staggered fashion, à la Calvo (1983). Due to price stickiness and the existence of nominal government debt, monetary policy has real effects. Monetary policy is described by a Taylor (1993)-type interest rate rule; fiscal policy is described by rules according to which the income tax rate and government purchases are set as functions of GDP and of the stock of real debt. The average ratio of government purchases to GDP, the average tax rate and the average debt-to-GDP ratio are calibrated to historical averages observed in OECD economies. The analysis takes these average values as given.

Sims' (2000) quadratic approximation method is used to solve the model, and to compute household welfare. The paper determines the response coefficients of the policy rules that maximize household welfare. Optimized monetary policy has a strong anti-inflation stance; optimized fiscal policy implies that the income tax rate is countercyclical, and that government purchases are procyclical; this result does not hinge on the degree of price stickiness--it also holds when prices are fully flexible. The procyclicality of government

¹ See also Chamley (1986) Aiyagari (1994), Chari et al. (1994), Aiyagari et al. (2001), Correia et al. (2001) and Schmitt-Grohé and Uribe (2001) (several of these papers also determine optimal monetary policy).

² The key technical difficulty is that Ramsey problems are (in general) not concave programming problems: the choice sets defined by the implementability conditions are, in general, not convex. Nevertheless, existing analyses generally cast the Ramsey problem as a Lagrange problem, and focus on the associated first-order conditions. No evidence is provided that the relevant second-order conditions hold.

Also, the literature suggests that the solution of the Ramsey problem may imply that government debt and tax rates are non-stationary, or nearly non-stationary (see e.g. Aiyagari et al. (2001) and Schmitt-Grohé and Uribe (2001)); this would imply that standard numerical techniques (namely techniques based on local approximations of dynamic systems around a deterministic steady state) would not be well suited for studying the quantitative properties of these solutions.

purchases reflects the fact that the household values government purchases: when GPD rises (due to a positive productivity shock), the household wishes to achieve a higher level of private consumption and of government consumption, and as a result the optimized fiscal policy rule prescribes an increase in government purchases. Because of monopolistic distortions in goods markets and a positive income tax, consumption is inefficiently low in the economy considered here (compared to the first-best Pareto efficient allocation). It appears that making the income tax rate countercyclical allows the government to increase *mean* consumption--and thus to bring consumption closer to its efficient level.

Kim and Kim (2001) have also explored welfare maximizing "simple" fiscal policy rules, using calibrated dynamic general equilibrium models. These authors consider a two-country world with flexible prices, and their analysis focuses on issues of international policy coordination; their analysis abstracts from government debt. By contrast, the paper here considers a closed economy with sticky prices and government debt.

Section 2 of the paper presents the model. Section 3 presents the results and Section 4 concludes.

2. The model

A closed economy with a representative household, firms, a monetary authority and a fiscal authority is considered (the structure of preferences, technologies and markets resembles that of Kollmann's (2001,2002,2003) open economy models). There is a single final good that is produced by combining a continuum of intermediate goods indexed by $s \in [0,1]$. The final good is produced by perfectly competitive firms, it can be consumed and used for investment. There is monopolistic competition in intermediate goods markets--each intermediate good is produced by a single firm. Intermediate goods producers use capital and labor as inputs. The household owns all producers and the capital stock, which it rents to producers. It also supplies labor. The markets for rental capital and for labor are competitive.

2.1. Final good production

The final good is produced using the aggregate technology

$$Z_t = \left\{ \int_0^1 q_t(s)^{(\nu-1)/\nu} ds \right\}^{\nu/(\nu-1)} \quad (1)$$

with $\nu > 1$, where $q_t(s)$ is the quantity of the type s intermediate good. Let $p_t(s)$ be the nominal price of that good. Cost minimization in final good production implies:

$$q_t(s) = (p_t(s) / P_t)^{-\nu} Z_t, \quad \text{with } P_t = \left\{ \int_0^1 p_t(s)^{1-\nu} ds \right\}^{1/(1-\nu)}. \quad (2)$$

Perfect competition in the final good market implies that the good's price is P_t (its marginal cost is $\left\{ \int_0^1 p_t(s)^{1-\nu} ds \right\}^{1/(1-\nu)}$).

2.2. Intermediate goods firms

The technology of the firm that produces intermediate good s is:

$$y_t(s) = \theta_t K_t(s)^\psi L_t(s)^{1-\psi}, \quad 0 < \psi < 1. \quad (4)$$

$y_t(s)$ is the firm's output at date t . $K_t(s)$ and $L_t(s)$ are the amounts of capital and labor used by the firm. θ_t is an exogenous productivity parameter that is identical for all intermediate goods producers. θ_t follows this process:

$$\theta_t = (1 - \rho^\theta) + \rho^\theta \theta_{t-1} + \varepsilon_t^\theta, \quad 0 \leq \rho^\theta < 1, \quad (5)$$

where ε_t^θ is a white noise with standard deviation σ^θ .

Let R_t and W_t be the rental rate of capital and the wage rate. Cost minimization implies: $L_t(s)/K_t(s) = \psi^{-1}(1-\psi)R_t/W_t$. The firm's marginal cost is: $MC_t = (1/\theta_t)R_t^\psi W_t^{1-\psi} \psi^{-\psi}(1-\psi)^{\psi-1}$. Demand for the firm's output is given by (3). Its profit is:

$$\pi_t(p_t(s)) = (p_t(s) - MC_t)(p_t(s)/P_t)^{-\nu} Z_t. \quad (7)$$

The representative household receives the profits of intermediate goods firms--at the household level, period t profits are taxed at the rate τ_t (see below).

There is staggered price setting, à la Calvo (1983): intermediate goods firms cannot change prices, in buyer currency, unless they receive a random "price-change signal." The probability of receiving this signal in any particular period is $1-d$, a constant. Thus, the mean price-change-interval is $1/(1-d)$. Following Yun (1996) and Erceg et al. (2000) it is assumed that when a firm does not receive a "price-change signal," its price is automatically increased at the steady state growth factor of the price level. (Throughout this paper, the term "steady state" refers to the deterministic steady state.) Firms are assumed to meet all demand at posted prices.

Consider an intermediate good producer that, at time t , sets a new price, $p_{t,t}$. If no "price-change signal" is received between t and $t+j$, the price is $p_{t,t}\Pi^j$ at $t+j$, where Π is the steady state growth factor of the final good price, P_t . The firm sets $p_{t,t} = \text{Arg Max}_p \sum_{j=0}^{\infty} d^j E_t \{ \rho_{t,t+j} (1-\tau_{t+j}) \pi_{t+j}(p\Pi^j) / P_{t+j} \}$, where $\rho_{t,t+\tau}$ is a pricing kernel (for valuing date $t+j$ pay-offs) that equals the household's marginal rate of substitution between consumption at t and at $t+j$ (see discussion below). Let $\Xi_{t,t+j} = \rho_{t,t+j} (P_{t+j})^{\nu-1} Z_{t+j}$. The solution of the maximization problem regarding $p_{t,t}$ is:

$$p_{t,t} = (\nu/(\nu-1)) \left\{ \sum_{j=0}^{\infty} (d\Pi^{-\nu})^j E_t \Xi_{t,t+j} (1-\tau_{t+j}) MC_{t+j} \right\} / \left\{ \sum_{j=0}^{\infty} (d\Pi^{1-\nu})^j E_t \Xi_{t,t+j} (1-\tau_{t+j}) \right\}. \quad (8)$$

The final good price P_t evolves according to:

$$(P_t)^{1-\nu} = d(P_{t-1}\Pi)^{1-\nu} + (1-d)(p_{t,t})^{1-\nu}. \quad (9)$$

2.3. The representative household

Household preferences are described by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t, G_t). \quad (12)$$

E_t denotes the mathematical expectation conditional upon complete information pertaining to period t and earlier. C_t and L_t are period t household consumption, labor effort and government purchases of the final good. $0 < \beta < 1$ is the subjective discount factor. U is a utility function given by:

$$U(C_t, L_t, G_t) = \ln(C_t) - L_t + \Psi \ln(G_t), \quad (13)$$

where Ψ is a parameter.

As indicated earlier, the household owns all domestic producers and accumulates physical capital. The law of motion of the capital stock is:

$$K_{t+1} = K_t(1-\delta) + I_t, \quad (14)$$

where I_t is gross investment, while $0 < \delta < 1$ is the depreciation rate of capital. The household also trades in nominal one-period bonds. The household pays taxes, T_t , to the

government. The tax is a proportional tax on labor income on "entrepreneurial" income, with tax rate τ_t : $T_t = \tau_t (W_t L_t + \int_0^1 \pi(p_t(s)) ds + R_t K_t - dP_t K_t)$. $W_t L_t$ is the household's labor income; $\int_0^1 \pi(p_t(s)) ds$ is the aggregate profit of intermediate goods producers, and $R_t K_t - dP_t K_t$ is income from capital rental (net of a depreciation allowance).

The household's period t budget constraint is:

$$A_{t+1} + P_t(C_t + I_t) = A_t(1 + i_{t-1}) + W_t L_t + \int_0^1 \pi(p_t(s)) ds + R_t K_t - T_t. \quad (15)$$

A_t is the net stock of bonds that mature in period t , while i_{t-1} is the interest rates on these bonds.

The household chooses a strategy $\{A_{t+1}, K_{t+1}, C_t, L_t\}_{t=0}^{t=\infty}$ to maximize its expected lifetime utility (12), subject to constraints (14) and (15) and to initial values A_0, K_0 . Ruling out Ponzi schemes, the following equations are first-order conditions of this decision problem:

$$1 = (1 + i_t) E_t \{ \rho_{t,t+1} (P_t / P_{t+1}) \}, \quad (16)$$

$$1 = E_t \{ \rho_{t,t+1} ([R_{t+1}/P_{t+1} - \delta](1 - \tau_{t+1}) + 1) \}, \quad (17)$$

$$(1 - \tau_t) W_t / P_t = C_t, \quad (18)$$

where $\rho_{t,t+1} = \beta C_t / C_{t+1}$. (16)-(17) are Euler conditions, and (18) says that the household equates its marginal rate of substitution between consumption and leisure to the after-tax real wage rate.

2.4. The government

The government purchases G_t units of the final good, in period t . Its budget constraint is:

$$P_t G_t + D_t(1 + i_{t-1}) = D_{t+1} + T_t, \quad (19)$$

where D_t is the net stock of government debt that matures in period t .

2.5. Market clearing conditions

Supply equals demand in intermediate goods markets because intermediate goods firms meet all demand at posted prices. Market clearing for the final good, labor, and rental capital requires:

$$Z_t = C_t + I_t + G_t, \quad L_t = \int_0^1 L_t(s) ds, \quad K_t = \int_0^1 K_t(s) ds, \quad (21)$$

where Z_t , L_t and K_t are the supplies of the final good, of labor, and of rental capital, respectively, while $\int_0^1 L_t(s) ds$ and $\int_0^1 K_t(s) ds$ represent total demand for labor and capital (by intermediate goods producers).

Market clearing for bonds requires:

$$A_t = D_t. \quad (22)$$

2.6. Policy rules

Much recent research has focused on monetary policy rules that stipulate a response of the interest rate to inflation (e.g., Taylor, 1993a, 1999). This paper considers the following interest rate rule:

$$i_t = i + \Gamma_i^\pi \hat{\Pi}_t, \quad (28)$$

with $\hat{\Pi}_t = (\Pi_t - \Pi)/\Pi$, where $\Pi_t = P_t/P_{t-1}$ is the gross final good inflation rate. i is the steady state nominal interest rate, and (as defined earlier) Π is the steady state gross inflation rate. Throughout the paper, variables without time subscripts denote steady state values, and $\hat{x}_t = (x_t - x)/x$ is the relative deviation of a variable x_t from its steady state value, x . Γ_π is a parameter.

Following the prior literature on fiscal policy rules, I assume that the income tax rate and government purchases (normalized by steady state GDP, Y) are set as functions of GDP (Y_t) and of the (real) stock of government debt:

$$\tau_t = \sigma + \Gamma_\tau^Y \hat{Y}_t + \Gamma^D i (D_t/P_t - d)/Y, \quad (29)$$

$$G_t/Y = \gamma + \Gamma_G^Y \hat{Y}_t - \Gamma^D i (D_t/P_t - d)/Y, \quad (30)$$

where d is the steady state value of real government debt (D_t/P). $\sigma, \gamma, \Gamma_\tau^Y, \Gamma^D, \Gamma_G^Y$, and Γ_π^D are parameters. Selecting $\Gamma^D > 0.5$ allows to ensure that government solvency conditions are satisfied.

The monetary and fiscal authorities make a commitment to set the coefficients of these policy rules at time-invariant values that maximize household welfare (subject to a restriction discussed below).

Two welfare criteria are considered:

(i) The expected value of household life-time utility, *conditional* on the state of the economy in an "initial" state (defined below).

(ii) The unconditional expected value of household life-time utility, $(1 - \beta)^{-1} E\{U(C_t, L_t, G_t)\}$ (i.e. mean welfare when the economy has reached its stochastic steady state).

The unconditional welfare criterion has widely been used in the literature.³ However, as discussed, for example, by Kim et al. (2002) this criterion is not optimal when households discount future instantaneous utility;⁴ these authors thus advocate the use of a conditional welfare criterion. The baseline model discussed below assumes hence that the policy authority maximizes conditional welfare (the unconditional welfare criterion is used in a sensitivity analysis). I consider a conditional welfare measure that pertains to an "initial" period in which the predetermined state variables and the productivity innovations equal their (deterministic) steady state values, and in which the tax rate and the ratio of government purchases to GDP take values that correspond to historical values (in OECD economies).

As stressed by, i.a., Lucas (1990), the current tax system (in OECD economies) is inefficient: Lucas (1990) shows that there is scope for drastic changes in tax rates that induce substantial increases in consumption, capital, GDP and welfare.

The solution method used here is based on a *local* approximation of the model, around a deterministic steady state. This method is thus not suitable for analyzing sizable tax reforms à la Lucas (1990)--in particular, it is not suited for capturing the transition dynamics from the current (inefficient) tax system to the new post-reform steady state.

The analysis here focuses thus on changes in fiscal policy rules that alter the *cyclical* behavior of the tax rate and of government purchases, but that do *not* affect their mean values. Specifically, I impose the restriction that the unconditional expected values of the tax rate and of the ratio of government purchases to GDP have to equal these exogenous constant values (that policy cannot alter):

³ See, e.g., Clarida et al. (1999), and Rotemberg and Woodford (1997) (who justify this criterion by pointing out that it "is not subject to any problem of time consistency"; p.70).

⁴ As pointed out by Levin (2002), the logic for this is analogous to the suboptimality of the Golden Rule for capital accumulation, compared to the Modified Golden Rule.

$$E\tau_t = 0.25, \quad E(G_t/Y_t) = 0.20. \quad (31)$$

I set the tax rate at 0.25 and the government to GDP ratio at 0.20 in the "initial" (deterministic) steady state that is considered for the computation of the conditional welfare measure; the implied ratio of government debt to annual GDP (in the initial steady state) is 0.50.

The decision problem of the monetary and fiscal authorities is to select the policy parameters $\Gamma_i^\pi, \Gamma_G^Y, \Gamma_\tau^Y, \Gamma^D, \sigma, \gamma$ that maximize the welfare criterion subject to the laws of motion of the endogenous variables implied by (1)-(30), and subject to (31).

2.7. Solution method and welfare computation

The model is solved using Sims' (2000) algorithm/computer code that is based on second-order Taylor expansions of the equilibrium conditions, around a (deterministic) steady state.⁵ I numerically maximize the objective function of the policy authorities (attention is restricted to parameter values for which a unique stationary equilibrium exists). Note that, in steady state, the tax rate and government purchases equal the intercepts of (29)-(30). As these intercepts are choice variables for the government, fiscal policy affects the steady state. The computation of the conditional welfare measure uses *two* steady states: (i) the initial steady state, in which the tax rate and the G/Y ratio are exogenously set at 0.25 and 0.20, respectively; (ii) the (deterministic) steady state induced by the policy parameters σ, γ that maximize (conditional) welfare.

Let x^I and x^{II} denote the values of a variables x_t in the deterministic steady state defined in (i) and in (ii), respectively. (By assumption: $\tau^I=0.25$, $G^I/Y^I=0.20$.) Welfare is computed by taking a second order approximation around the steady state (ii), taking as given the values of the predetermined variables (capital, debt etc) in steady state (i) as initial values.

2.8. Parameters (non-policy)

The model is calibrated to quarterly data. The steady state *real* interest rates r is set at $r=0.01$, a value that corresponds roughly to the long-run average (quarterly) return on capital. The subjective discount factor is, hence, set at $1/(1.01)$, since $\beta(1+r)=1$ holds in steady state. The weight of government consumption in the household's utility function is set at $\Psi=0.2$ (this value implies that, in a first-best efficient allocation, government purchases would represent 20% of private consumption).

The steady state price-marginal cost markup factor for intermediate goods is set at $\nu/(\nu-1)=1.2$, consistent with the findings of Martins et al. (1996) for the US and for European countries. The technology parameter ψ (see (4)) is set at $\psi=0.24$, which entails a 60% steady state labor income/GDP ratio, consistent with US and European data. Aggregate data suggest a quarterly capital depreciation rate of about 2.5%; thus, $\delta=0.025$ is used.

Estimates of Calvo-style price setting equations for the US and for European countries suggest that the average price-change interval is about 4 quarters (e.g., Lopez-Salido (2000)). Hence, d is set at $d=0.75$. The steady state growth factors of the price levels is set at $\Pi=1$ (Π has no effect on real variables, because of indexing); thus the steady state nominal interest rate is $i=r=0.01$.

The autocorrelation of productivity and the standard deviation of productivity innovations are set at values that are standard in the RBC literature: $\rho^\theta = 0.95$, $\sigma^\theta = 0.01$.

⁵ See Kim et al. (2002) and Kollmann (2002a) for a more detailed discussion of the Sims algorithm. Guu and Judd (1993), Gaspar and Judd (1996), Kim and Kim (1999), Collard and Juillard (2001), Schmitt-Grohé and Uribe (2001b) and Anderson and Levin (2002) also develop solutions of dynamic models based on second-order expansions.

The steady state tax rate is set at $\tau = 0.25$ (which implies that the steady state ratio of tax revenues to GDP is 0.22); steady state government purchases are set in such a way that the steady state ratio of government purchases to GDP is 0.20. The implied steady state ratio of government debt to annual GDP is 0.50.

3. Results

The results are reported in Tables 1 and 2. Table 1 shows predicted standard deviations, (auto-)correlations, and mean values of key variables, while Table 2 shows impulse responses to a 1% productivity innovation.

The variables are quarterly. $Def_t = (B_{t+1} - B_t)/PY$ and $Debt_t = D_{t+1}/PY$ are the fiscal deficit and government debt, in real terms, normalized by steady state GDP. The statistics for the interest rate (i_t), the tax rate (τ_t), the fiscal deficit (Def_t), and government debt ($Debt_t$) refer to differences of these variables from steady state values (i_t is a quarterly rate expressed in fractional units), while statistics for the remaining variables refer to relative deviations from steady state values. All statistics are expressed in percentage terms. Table 1 also reports welfare; welfare is expressed as the permanent relative change in consumption, compared to the steady state, that yields the (un-)conditional expected welfare in the stochastic equilibrium: the measure of unconditional welfare, ζ^u , is defined as $U((1 + \zeta^u)C, L, G) = E\{U(C_t, L_t, G_t)\}$; the measure of conditional welfare, ζ^v , is defined as $(1 - \beta)^{-1} U((1 + \zeta^v)C, L, G) = E_0\{\sum_{t=0}^{\infty} U(C_t, L_t, G_t)\}$.

The last 8 rows of Table 1 reports the optimized policy parameters. $d\sigma$ and $d\gamma$ denote the differences between the intercepts of the fiscal policy rules (29),(30) that maximize unconditional welfare, and the values of these intercepts in the "initial" steady state (see discussion in Sect. 2.6. and 2.7.).

Cols. 1 and 2 of Table 1 assume that the government can issue one-period bonds; Col. 1 assumes that prices are sticky (baseline model), while Col. 2 assumes flexible prices. Cols. 3 and 4 assume that governments do not issue bonds--instead, a lump sum tax is used to cover the difference between the revenue from the income tax and government purchases; Col. 3 assumes sticky prices, while Col. 4 assumes flexible prices.

Col. 5 reports predictions for the first-best equilibrium of this economy--this equilibrium represents the solution of a social planning problem in which household welfare is maximized subject to the resource constraints of this economy.⁶ (Welfare in the stochastic first-best equilibrium is expressed as a permanent equivalent variation in consumption, relative to the deterministic steady state of the undistorted economy.)

Note that monetary policy has no real effects in the flex-prices economy without government bonds, as well as in the first-best economy. In these two economies, changes in the monetary policy response coefficient Γ_i^π have thus no effect on real variables and on welfare; I set $\Gamma_i^\pi = 1.5$ in these two economies.

The most surprising result from Table 1 is that optimized policy in the distorted economies entails a cyclical behavior of the key macro variables that is surprisingly close to the behavior displayed by the first-best (undistorted) economy (considered in Col. 5). Also, the cyclical behavior of the key real variables and their mean values are quite similar across the four model variants with tax distortions considered in Cols. 1-4.

⁶ The first-best equilibrium represents also the competitive equilibrium of a variant this economy (under flexible prices) in which the household directly selects government purchases.

In the variants of the model in which monetary policy has real effects, optimized monetary policy has a strict anti-inflation stance: the response coefficient of the interest rate to inflation is sizable (e.g. $\Gamma_i^\pi = 4.63$ in the sticky-prices baseline model with nominal government bonds). As a result, the standard deviation of inflation is low in these model variants (that standard deviation does not exceed 0.10%).

In the model variant with bonds, as well as in the variant with a lump sum tax, the income tax rate is countercyclical, and government purchases are procyclical ($\Gamma_\tau^Y < 0$, $\Gamma_G^Y > 0$). The tax rate is negatively correlated with GDP, while government purchases are positively correlated with GDP. Table 2 shows that the tax rate [government purchases] falls [rises], in response to a positive productivity innovation (of course, GDP responds positively to that innovation).

The fact that optimizing policy entails that government purchases are procyclical is not surprising: government purchases enter positively in the household utility function. A positive productivity shock raises household wealth; it thus raises the household's "demand" for government consumption.

The fact that optimizing policy entails that the income tax rate is countercyclical can be explained in the following way: the countercyclicity of the tax rate raises mean consumption and the mean capital stock; as consumption and capital are too low (relative to the first-best allocation), in this economy (due to the tax distortion, and due to monopolistic competition in the market for intermediate goods), this increase in mean consumption (and capital) raises welfare.

To understand why the countercyclicity of the tax rate raises mean consumption, note that (18) shows that, in equilibrium, consumption equals the after-tax real wage rate. The countercyclicity of the tax rate entails that $(1 - \tau_t)$ is positively correlated with the real wage rate, W_t/P_t . Holding constant the expected value of the real wage rate and of the tax rate, an increase in the covariance between $(1 - \tau_t)$ and W_t/P_t raises the expected value of $(1 - \tau_t)W_t/P_t$, and thus of C_t . (In fact, the mean real wage increases when the income tax rate is made procyclical [compared to a situation with an acyclical tax rate], as this raises the mean capital stock....[DISCUSSION TO BE ADDED]).

3. Conclusion

This paper has studied the welfare effects of monetary and fiscal policy rules, in a dynamic general equilibrium model with sticky prices. The model features capital accumulation and endogenous labor effort, and exogenous productivity shocks. Government purchases are valued positively by the private sector. These purchases are financed using a proportional income tax. The government issues nominal one-period bonds. Monetary policy is described by an interest rate rule; fiscal policy is described by rules according to which the income tax rate and government purchases are set as functions of GDP. Sims' (2000) quadratic approximation method is used to solve the model, and to compute household welfare. The paper determines the response coefficients of the policy rules that maximize household welfare. Optimized monetary policy has a strong anti-inflation stance; optimized fiscal policy implies that the income tax rate is countercyclical, and that government purchases are procyclical; this result does not hinge on the degree of price stickiness.

Table 1. Optimized policy rules and first best allocation

| | Deficits financed | | Lump sum tax | | First best allocation |
|---|-------------------|--------------|---------------|--------------|-----------------------|
| | using bonds | | | | |
| | Sticky Prices | Flex. Prices | Sticky Prices | Flex. Prices | |
| | (1) | (2) | (3) | (4) | (5) |
| Standard deviations (in %) | | | | | |
| Y | 6.70 | 7.09 | 7.68 | 7.64 | 6.10 |
| C | 5.37 | 5.68 | 5.96 | 5.90 | 4.22 |
| I | 23.66 | 25.42 | 28.82 | 28.42 | 20.05 |
| Π | 0.04 | 0.10 | 0.01 | 0.33 | 0.26 |
| i | 0.18 | 0.26 | 0.19 | 0.49 | 0.38 |
| G | 3.75 | 3.55 | 3.80 | 4.05 | 2.90 |
| τ | 0.60 | 0.81 | 1.00 | 0.97 | 0.00 |
| Debt | 17.40 | 10.31 | -- | -- | -- |
| Def | 0.52 | 0.38 | -- | -- | -- |
| Correlations with GDP | | | | | |
| i | -0.12 | -0.15 | 0.00 | -0.22 | -0.08 |
| G | 0.93 | 0.96 | 1.00 | 1.00 | 0.88 |
| τ | -0.89 | -0.97 | -1.00 | -1.00 | -- |
| Debt | -0.57 | -0.57 | -- | -- | -- |
| Def | -0.84 | -0.81 | -- | -- | -- |
| Autocorrelations | | | | | |
| Y | 0.94 | 0.94 | 0.94 | 0.94 | 0.93 |
| i | 0.93 | 0.93 | 0.93 | 0.95 | 0.93 |
| G | 0.97 | 0.96 | 0.94 | 0.94 | 0.99 |
| τ | 0.98 | 0.96 | 0.94 | 0.94 | 0.99 |
| Debt | 0.99 | 0.99 | 0.99 | -- | -- |
| Def | 0.81 | 0.72 | 0.93 | -- | -- |
| Means (in %) | | | | | |
| Y | 0.16 | 0.19 | 0.24 | 0.34 | 0.10 |
| C | 0.14 | 0.17 | 0.21 | 0.27 | 0.07 |
| L | -0.02 | -0.02 | -0.00 | -0.00 | -0.01 |
| K | 0.56 | 0.65 | 0.80 | 0.79 | 0.29 |
| G | -0.01 | -0.01 | -0.01 | -0.01 | 0.01 |
| τ | 0.00 | 0.00 | 0.00 | 0.00 | -- |
| Debt | -0.80 | -1.13 | -- | -- | -- |
| Def | 0.00 | 0.00 | -- | -- | -- |
| Welfare (% equivalent variation in consumption) | | | | | |
| ζ'' | -0.004 | 0.004 | 0.017 | 0.016 | -0.010 |
| ζ^c | -0.015 | -0.014 | -0.009 | -0.009 | -0.018 |

Table 1 - - continued

Policy parameters

| | | | | | |
|-----------------|--------|---------|---------|---------|--------|
| $d\sigma$ | 2.6e-4 | -2.8e-4 | 4.5e-4 | 4.4e-4 | -- |
| $d\gamma$ | 3e-5 | 6e-5 | -2.4e-4 | -2.3e-4 | -- |
| Γ_i^π | 4.63 | 2.45 | 17.97 | 1.5(*) | 1.5(*) |
| Γ_τ^Y | -0.05 | -0.09 | -0.13 | -0.12 | -- |
| Γ_G^Y | 0.07 | 0.08 | 0.10 | 0.10 | -- |
| Γ^D | 1.87 | 2.45 | -- | -- | -- |

Table 2. % responses to 1 standard deviation productivity innovations

| | <i>Y</i> | <i>C</i> | <i>I</i> | <i>L</i> | <i>K</i> | <i>P</i> | <i>i</i> | <i>G</i> | τ | <i>Debt</i> | θ |
|--|----------|----------|----------|----------|----------|----------|----------|----------|--------|-------------|----------|
| (a) Sticky prices; deficits financed using bonds | | | | | | | | | | | |
| $\tau=0$ | 2.14 | 0.82 | 10.96 | 1.50 | 0.27 | 0.01 | 0.06 | 0.83 | -0.11 | -0.30 | 1.00 |
| $\tau=4$ | 1.69 | 0.97 | 6.87 | 0.87 | 1.04 | 0.04 | 0.02 | 0.72 | -0.10 | -0.83 | 0.81 |
| $\tau=24$ | 0.60 | 0.73 | 0.15 | -0.04 | 1.39 | -0.03 | -0.03 | 0.43 | -0.07 | -2.12 | 0.29 |
| $\tau=100$ | 0.06 | 0.06 | -0.02 | 0.03 | 0.13 | -0.21 | -0.00 | 0.11 | -0.00 | -0.94 | 0.01 |
| (b) Flexible prices; deficits financed using bonds | | | | | | | | | | | |
| $\tau=0$ | 2.30 | 0.88 | 11.75 | 1.71 | 0.29 | 0.03 | 0.08 | 0.93 | -0.21 | -0.25 | 1.00 |
| $\tau=4$ | 1.81 | 1.04 | 7.39 | 1.01 | 1.12 | 0.10 | 0.02 | 0.78 | -0.18 | -0.53 | 0.81 |
| $\tau=24$ | 0.61 | 0.78 | 0.11 | -0.05 | 1.48 | -0.11 | -0.04 | 0.39 | -0.08 | -1.32 | 0.29 |
| $\tau=100$ | 0.04 | 0.05 | -0.04 | 0.01 | 0.10 | -0.60 | -0.00 | 0.07 | -0.01 | -0.46 | 0.02 |
| (c) Sticky prices; deficits financed using lump sum taxes | | | | | | | | | | | |
| $\tau=0$ | 2.68 | 0.94 | 13.72 | 2.21 | 0.34 | 0.00 | 0.07 | 1.34 | -0.34 | -- | 1.00 |
| $\tau=4$ | 2.05 | 1.13 | 8.36 | 1.27 | 1.28 | 0.01 | 0.02 | 1.03 | -0.26 | -- | 0.81 |
| $\tau=24$ | 0.58 | 0.81 | -0.15 | -0.13 | 1.59 | -0.01 | -0.03 | 0.29 | -0.07 | -- | 0.29 |
| $\tau=100$ | 0.01 | 0.01 | -0.06 | -0.01 | 0.05 | -0.05 | -0.00 | 0.00 | -0.00 | -- | 0.01 |
| (d) Flexible prices; deficits financed using lump sum taxes | | | | | | | | | | | |
| $\tau=0$ | 2.65 | 0.93 | 13.47 | 2.17 | 0.33 | 0.09 | 0.13 | 1.41 | -0.33 | -- | 1.00 |
| $\tau=4$ | 2.03 | 1.12 | 8.24 | 1.26 | 1.26 | 0.25 | 0.03 | 1.08 | 0.03 | -- | 0.81 |
| $\tau=24$ | 0.59 | 0.81 | -0.13 | -0.12 | 1.57 | -0.60 | -0.08 | 0.31 | -0.08 | -- | 0.29 |
| $\tau=100$ | 0.01 | 0.02 | -0.06 | -0.01 | 0.05 | -2.04 | -0.00 | 0.01 | -0.00 | -- | 0.01 |
| (e) First best allocation | | | | | | | | | | | |
| $\tau=0$ | 2.20 | 0.61 | 9.92 | 1.59 | 0.24 | 0.08 | 0.13 | 0.61 | -- | -- | 1.00 |
| $\tau=4$ | 1.64 | 0.80 | 5.74 | 0.84 | 0.91 | 0.25 | 0.03 | 0.80 | -- | -- | 0.81 |
| $\tau=24$ | 0.43 | 0.58 | -0.25 | -0.14 | 1.02 | -0.38 | -0.06 | 0.58 | -- | -- | 0.29 |
| $\tau=100$ | 0.01 | 0.02 | -0.03 | -0.01 | 0.03 | -1.41 | -0.00 | 0.01 | -- | -- | 0.01 |

Notes: τ : periods after shock. Columns labeled *Y*, *C*, etc. show responses of the corresponding variables. *P*: price of final good; the remaining variables are defined in Table 1.

The impulse responses are generated as follows. At a given date, say *T*, all state variables are set at steady state values. A "baseline" path for the endogenous variables is computed by setting all exogenous innovations to zero in periods $t \geq T$. Then responses to one-time 1 standard deviation exogenous innovation at *T* are computed; the Table reports differences/relative deviations (that have been multiplied by 100, i.e. expressed in percentage terms) of these responses from the "baseline" path; responses of interest rates (*i*) and *Debt*: differences from baseline path (*Debt*: in real terms and normalized by steady state GDP); responses of remaining variables: relative deviations from baseline path.